

Homework 6

Work in teams of two. Homework is due every two weeks.

Submission format: Host your solution in a private Git repository on one of the following platforms and add Rouven as a collaborator: IBR GitLab (**kniep**), TU GitLab (**r.kniep**), GitHub (**rkniep**), or Codeberg (**rkniep**). If you want to use a different platform, please ask first.

Before the deadline, send the link to your submission commit by e-mail to **kniep@ibr.cs.tu-bs.de**. A single PDF with your written answers is sufficient for this sheet.

This sheet is an **exam-preparation sheet**. Every task below is drawn verbatim from the exam question catalogue, so working through them now is the most direct preparation you can do. Answer in **your own words** and hand in written answers (a single PDF is enough). Keep answers concise but complete: a good answer is one you could reproduce under exam conditions.

Not every question will appear in the exam, and the exam may ask slight variants (reworded for precision, or — for the modeling and CDCL tasks — with a different problem or input). The point of this sheet is that you have prepared the underlying ideas, not memorized these specific instances.

Exercise 1 (CPU Architecture & Performance Engineering): (3+2+3+3 points)

- a) Describe the terms ‘array of structures’ and ‘structure of arrays’ in your own words and the benefits and drawbacks of each.
- b) When solving hard problems, what simple approach to parallelization can often be surprisingly (and embarrassingly) competitive with complex, parallelized solvers?
- c) State Amdahl’s Law and explain what it implies for where you should spend optimization effort.
- d) Profiling tools fall into two big families. Name both, describe each mechanism, and give the main trade-off.

Exercise 2 (Data Structures & Optimization Fundamentals): (4+3 points)

- a) State the theoretical amortized complexity of Union-Find (**find/unite**) with union-by-rank + path compression, and the complexity to assume in practice. Justify.

- b) Why can heuristics that find good solutions be a valuable addition to branch & bound-based algorithms?

Exercise 3 (Declarative Modeling): **(7 points)**

Read the problem statement, then apply the five-component recipe — for each component extract the relevant phrase(s) and write the corresponding formal object.

A delivery company must assign every parcel from a given batch to exactly one of its available vans. Each parcel has a known weight, and each van has a fixed weight capacity that the total weight of the parcels loaded into it must not exceed. Loading a parcel into a particular van incurs a handling cost that depends on the parcel–van pair. The company wants to load every parcel while spending as little as possible on handling.

Exercise 4 (LP & MIP): **(4+4 points)**

- a) What are the benefits of the barrier interior point algorithm over simplex? What are the benefits of the simplex algorithm? Why can both be a valuable part of a MIP solver and even both be used in solving the same MIP?
- b) State the general Big-M construction for gating a linear constraint by a binary. Then explain the main practical drawback of the Big-M method and the mitigations.

Exercise 5 (Graph & Network Algorithms and Routing): **(3+4 points)**

- a) Time-dependent shortest paths (edge travel time depends on departure time) can still be solved with Dijkstra. Which condition must hold, and what does it mean intuitively?
- b) Vehicle Routing (VRP): give an informal description of the core problem, then name at least three constraints/objective variants that occur in practice (restrict to those in the lecture).

Exercise 6 (SAT & CP-SAT): **(7+3 points)**

Notation: we write 1 instead of x_1 and $\bar{1}$ instead of $\bar{x}_1$, and 123 instead of $1 \vee 2 \vee 3$.

- a) Perform CDCL on the following formula until you have run into two conflicts or found the formula to be satisfied or unsatisfiable. When making a decision, prioritize variables with lower indices over those with higher indices, and set the variable to false in your decision. To make intermediate steps readable, fill out a new trail after each conflict.

$$1\bar{2}, 123, \bar{1}\bar{2}3, 12\bar{3}4, \bar{3}4, \bar{1}\bar{2}4, \bar{1}2$$

- b) Why is branching by the VSIDS (SAT decision) heuristic alone not sufficient to reach a complete solution in CP-SAT? What mechanism closes the gap?